



Kapak Konusu: İntegraller II

İntegral Formülleri

Bazı basit ifadeler

1. $\int d f(x) = \int f'(x) dx = f(x) + C$
2. $\int a du = a \int du$
3. $\int (du \pm dv \pm dw \pm \dots) = \int du \pm \int dv \pm \int dw \pm \dots$
4. $\int u^n du = \frac{u^{n+1}}{n+1} + C$
5. $\int \frac{du}{u} = \ln u + C$

 $a + bu$ içeren rasyonel ifadeler

6. $\int (a + bu)^n du = \frac{(a + bu)^{n+1}}{b(n+1)} + C$
7. $\int \frac{du}{a + bu} = \frac{1}{b} \ln(a + bu) + C$
8. $\int \frac{u du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln(a + bu)) + C$
9. $\int \frac{u^2 du}{a + bu} = \frac{1}{b^3} \left(\frac{1}{2} (a + bu)^2 - 2a(a + bu) + a^2 \ln(a + bu) \right) + C$
10. $\int \frac{u du}{(a + bu)^2} = \frac{1}{b^2} \left(\frac{a}{a + bu} + \ln(a + bu) \right) + C$
11. $\int \frac{u^2 du}{(a + bu)^2} = \frac{1}{b^3} \left(a + bu - \frac{a^2}{a + bu} - 2a \ln(a + bu) \right) + C$
12. $\int \frac{u du}{(a + bu)^3} = \frac{1}{b^2} \left(-\frac{1}{a + bu} + \frac{a}{2(a + bu)^2} \right) + C$
13. $\int \frac{du}{u(a + bu)} = -\frac{1}{a} \ln \left(\frac{a + bu}{u} \right) + C$
14. $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left(\frac{a + bu}{u} \right) + C$
15. $\int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left(\frac{a + bu}{u} \right) + C$

 $a^2 \pm b^2 u^2$ içeren ifadeler

16. $\int \frac{du}{a^2 + b^2 u^2} = \frac{1}{ab} \arctan \frac{bu}{a} + C$
- 17a. $\int \frac{du}{a^2 - b^2 u^2} = \frac{1}{2ab} \ln \left(\frac{a + bu}{a - bu} \right) + C \quad (a^2 > b^2 u^2)$

$$17 \text{ b. } \int \frac{du}{b^2u^2 - a^2} = \frac{1}{2ab} \ln \left(\frac{bu - a}{bu + a} \right) + C \quad (a^2 < b^2u^2)$$

$$18. \int u(a^2 \pm b^2u^2)^n du = \frac{(a^2 \pm b^2u^2)^{n+1}}{\pm 2b^2(n+1)} + C \quad (n \neq -1)$$

$$19. \int \frac{udu}{a^2 \pm b^2u^2} = \frac{1}{\pm 2b^2} \ln(a^2 \pm b^2u^2) + C$$

$$20. \int \frac{u^m du}{(a^2 \pm b^2u^2)^p} = \frac{u^{m-1}}{\pm b^2(m-2p+1)(a^2 \pm b^2u^2)^{p-1}} - \frac{a^2(m-1)}{\pm b^2(m-2p+1)} \int \frac{u^{m-2} du}{(a^2 \pm b^2u^2)^p}$$

$$21. \int \frac{u^m du}{(a^2 \pm b^2u^2)^p} = \frac{u^{m+1}}{2a^2(p-1)(a^2 \pm b^2u^2)^{p-1}} - \frac{m-2p+3}{2a^2(p-1)} \int \frac{u^m du}{(a^2 \pm b^2u^2)^{p-1}}$$

$$22. \int \frac{du}{u(a^2 \pm b^2u^2)} = \frac{1}{2a^2} \ln \left(\frac{u^2}{a^2 \pm b^2u^2} \right) + C$$

$$23. \int \frac{du}{u^m(a^2 \pm b^2u^2)^p} = -\frac{1}{a^2(m-1)u^{m-1}(a^2 \pm b^2u^2)^{p-1}} - \frac{\pm b^2(m+2p-3)}{a^2(m-1)} \int \frac{du}{u^{m-2}(a^2 \pm b^2u^2)^p}$$

$$24. \int \frac{du}{u^m(a^2 \pm b^2u^2)^p} = \frac{1}{2a^2(p-1)u^{m-1}(a^2 \pm b^2u^2)^{p-1}} + \frac{m+2p-3}{2a^2(p-1)} \int \frac{du}{u^m(a^2 \pm b^2u^2)^{p-1}}$$

$\sqrt{a+bu}$ içeren ifadeler

$$25. \int u\sqrt{a+bu} du = -\frac{2(2a-3bu)(a+bu)^{\frac{3}{2}}}{15b^2} + C$$

$$26. \int u^2\sqrt{a+bu} du = -\frac{2(8a^2-12abu+15b^2u^2)(a+bu)^{\frac{3}{2}}}{105b^3} + C$$

$$27. \int u^m\sqrt{a+bu} du = \frac{2u^m(a+bu)^{\frac{3}{2}}}{b(2m+3)} - \frac{2am}{b(2m+3)} \int u^{m-1}\sqrt{a+bu} du$$

$$28. \int \frac{u du}{\sqrt{a+bu}} = -\frac{2(2a-bu)\sqrt{a+bu}}{3b^2} + C$$

$$29. \int \frac{u^2 du}{\sqrt{a+bu}} = \frac{2(8a^2-4abu+3b^2u^2)\sqrt{a+bu}}{15b^3} + C$$

$$30. \int \frac{u^m du}{\sqrt{a+bu}} = \frac{2u^m\sqrt{a+bu}}{b(2m+1)} - \frac{2am}{b(2m+1)} \int \frac{u^{m-1} du}{\sqrt{a+bu}}$$

$$31. \int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}} \ln \left(\frac{\sqrt{a+bu} - \sqrt{a}}{\sqrt{a+bu} + \sqrt{a}} \right) + C \quad \text{eğer } a > 0 \text{ ise}$$

$$32. \int \frac{du}{u\sqrt{a+bu}} = \frac{2}{\sqrt{-a}} \arctan \sqrt{\frac{a+bu}{-a}} + C \quad \text{eğer } a < 0 \text{ ise}$$

$$33. \int \frac{du}{u^m\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(m-1)u^{m-1}} - \frac{b(2m-3)}{2a(m-1)} \int \frac{du}{u^{m-1}\sqrt{a+bu}}$$

$$34. \int \frac{\sqrt{a+bu} du}{u} = 2\sqrt{a+bu} + a \int \frac{du}{u\sqrt{a+bu}}$$

$$35. \int \frac{\sqrt{a+bu} du}{u^m} = -\frac{(a+bu)^{\frac{3}{2}}}{a(m-1)u^{m-1}} - \frac{b(2m-5)}{2a(m-1)} \int \frac{\sqrt{a+bu} du}{u^{m-1}}$$

$\sqrt{u^2 \pm a^2}$ içeren ifadeler

Bu formül grubunda

$\ln(u + \sqrt{u^2 + a^2})$ ifadesi $\arg \operatorname{sh} \frac{u}{a}$ ile

$\ln(u + \sqrt{u^2 - a^2})$ ifadesi $\arg \operatorname{ch} \frac{u}{a}$ ile

$\ln \frac{a + \sqrt{u^2 + a^2}}{u}$ ifadesi $\arg \operatorname{sh} \frac{a}{u}$ ile

değiştirilebilir.

$$36. \int (u^2 \pm a^2)^{\frac{1}{2}} du = \frac{u}{2} \sqrt{u^2 \pm a^2} \pm \frac{a^2}{2} \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$37. \int (u^2 \pm a^2)^{\frac{n}{2}} du = \frac{u(u^2 \pm a^2)^{\frac{n}{2}}}{n+1} \pm \frac{na^2}{n+1} \int (u^2 \pm a^2)^{\frac{n}{2}-1} du \quad (n \neq -1)$$

$$38. \int u(u^2 \pm a^2)^{\frac{n}{2}} du = \frac{(u^2 \pm a^2)^{\frac{n}{2}+1}}{n+2} + C \quad (n \neq -2)$$

$$39. \int u^m (u^2 \pm a^2)^{\frac{n}{2}} du = \frac{u^{m-1} (u^2 \pm a^2)^{\frac{n}{2}+1}}{n+m+1} - \frac{\pm a^2(m-1)}{n+m+1} \int u^{m-2} (u^2 \pm a^2)^{\frac{n}{2}} du$$

$$40. \int \frac{du}{(u^2 \pm a^2)^{\frac{1}{2}}} = \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$41. \int \frac{du}{(u^2 \pm a^2)^{\frac{3}{2}}} = \frac{u}{\pm a^2 \sqrt{u^2 \pm a^2}} + C$$

$$42. \int \frac{u du}{(u^2 \pm a^2)^{\frac{3}{2}}} = \frac{(u^2 \pm a^2)^{1-\frac{n}{2}}}{2-n} + C$$

$$43. \int \frac{u^2 du}{(u^2 \pm a^2)^{\frac{1}{2}}} = \frac{u}{2} \sqrt{u^2 \pm a^2} - \frac{\pm a^2}{2} \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$44. \int \frac{u^2 du}{(u^2 \pm a^2)^{\frac{n}{2}}} = -\frac{u}{\sqrt{u^2 \pm a^2}} + \ln(u + \sqrt{u^2 \pm a^2}) + C$$

$$45. \int \frac{u^m du}{(u^2 \pm a^2)^{\frac{n}{2}}} = -\frac{u^{m-1}}{(m-n+1)(u^2 \pm a^2)^{\frac{n}{2}-1}} - \frac{\pm a^2(m-1)}{m-n+1} \int \frac{u^{m-2} du}{(u^2 \pm a^2)^{\frac{n}{2}}}$$

$$46. \int \frac{u^m du}{(u^2 \pm a^2)^{\frac{n}{2}}} = \frac{u^{m+1}}{\pm a^2(n-2)(u^2 \pm a^2)^{\frac{n}{2}-1}} - \frac{m-n+3}{\pm a^2(n-2)} \int \frac{u^m du}{(u^2 \pm a^2)^{\frac{n}{2}-1}}$$

$$47. \int \frac{du}{u(u^2 + a^2)^{\frac{1}{2}}} = -\frac{1}{a} \ln \left(\frac{a + \sqrt{u^2 + a^2}}{u} \right) + C$$

$$48. \int \frac{du}{u(u^2 - a^2)^{\frac{1}{2}}} = \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C$$

$$49. \int \frac{du}{u^2(u^2 \pm a^2)^{\frac{1}{2}}} = -\frac{\sqrt{u^2 \pm a^2}}{\pm a^2 u} + C$$

$$50. \int \frac{du}{u^3(u^2 + a^2)^{\frac{1}{2}}} = -\frac{\sqrt{u^2 + a^2}}{2a^2 u^2} + \frac{1}{2a^3} \ln \left(\frac{a + \sqrt{u^2 + a^2}}{u} \right) + C$$

$$51. \int \frac{du}{u^3(u^2 - a^2)^{\frac{1}{2}}} = \frac{\sqrt{u^2 - a^2}}{2a^2 u^2} + \frac{1}{2a^3} \operatorname{arcsec} \frac{u}{a} + C$$

$$52. \int \frac{du}{u^m(u^2 \pm a^2)^{\frac{n}{2}}} = -\frac{1}{\pm a^2(m-1)u^{m-1}(u^2 \pm a^2)^{\frac{n}{2}-1}} - \frac{m+n-3}{\pm a^2(m-1)} \int \frac{du}{u^{m-2}(u^2 \pm a^2)^{\frac{n}{2}}}$$

$$53. \int \frac{du}{u^m(u^2 \pm a^2)^{\frac{n}{2}}} = \frac{1}{\pm a^2(n-2)u^{m-1}(u^2 \pm a^2)^{\frac{n}{2}-1}} + \frac{m+n-3}{\pm a^2(n-2)} \int \frac{du}{u^m(u^2 \pm a^2)^{\frac{n}{2}-1}}$$

$$54. \int \frac{(u^2 \pm a^2)^{\frac{1}{2}} du}{u} = \sqrt{u^2 + a^2} - a \ln \left(\frac{a + \sqrt{u^2 + a^2}}{u} \right) + C$$

$$55. \int \frac{(u^2 - a^2)^{\frac{1}{2}} du}{u} = \sqrt{u^2 - a^2} - a \operatorname{arcsec} \frac{u}{a} + C$$

$$56. \int \frac{(u^2 \pm a^2)^{\frac{1}{2}} du}{u^2} = -\frac{\sqrt{u^2 \pm a^2}}{u} + \ln \left(u + \sqrt{u^2 \pm a^2} \right) + C$$

$$57. \int \frac{(u^2 \pm a^2)^{\frac{n}{2}} du}{u^m} = -\frac{(u^2 \pm a^2)^{\frac{n}{2}+1}}{\pm a^2(m-1)u^{m-1}} - \frac{m-n-3}{\pm a^2(m-1)} \int \frac{(u^2 \pm a^2)^{\frac{n}{2}} du}{u^{m-2}}$$

$$58. \int \frac{(u^2 \pm a^2)^{\frac{n}{2}} du}{u^m} = \frac{(u^2 \pm a^2)^{\frac{n}{2}}}{(n-m+1)u^{m-1}} + \frac{\pm a^2 n}{n-m+1} \int \frac{(u^2 \pm a^2)^{\frac{n}{2}-1} du}{u^m}$$

$\sqrt{a^2 - u^2}$ içeren ifadeler

$$59. \int (a^2 - u^2)^{\frac{1}{2}} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsin \frac{u}{a} + C$$

60. $\int (a^2 - u^2)^{\frac{n}{2}} du = \frac{u(a^2 - u^2)^{\frac{n}{2}}}{n+1} + \frac{a^2 n}{n+1} \int (a^2 - u^2)^{\frac{n}{2}-1} du \quad (n \neq -1)$
61. $\int u(a^2 - u^2)^{\frac{n}{2}} du = -\frac{(a^2 - u^2)^{\frac{n}{2}+1}}{n+2} + C \quad (n \neq -2)$
62. $\int u^m (a^2 - u^2)^{\frac{n}{2}} du = -\frac{u^{m-1}(a^2 - u^2)^{\frac{n}{2}+1}}{n+m+1} + \frac{a^2(m-1)}{n+m+1} \int u^{m-2}(a^2 - u^2)^{\frac{n}{2}} du$
63. $\int \frac{du}{(a^2 - u^2)^{\frac{1}{2}}} = \arcsin \frac{u}{a} + C$
64. $\int \frac{du}{(a^2 - u^2)^{\frac{3}{2}}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$
65. $\int \frac{u du}{(a^2 - u^2)^{\frac{n}{2}}} = \frac{(a^2 - u^2)^{1-\frac{n}{2}}}{n-2} + C$
66. $\int \frac{u^2 du}{(a^2 - u^2)^{\frac{1}{2}}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \arcsin \frac{u}{a} + C$
67. $\int \frac{u^2 du}{(a^2 - u^2)^{\frac{3}{2}}} = \frac{u}{\sqrt{a^2 - u^2}} - \arcsin \frac{u}{a} + C$
68. $\int \frac{u^m du}{(a^2 - u^2)^{\frac{n}{2}}} = -\frac{u^{m-1}}{(m-n+1)(a^2 - u^2)^{\frac{n}{2}-1}} + \frac{a^2(m-1)}{m-n+1} \int \frac{u^{m-2} du}{(a^2 - u^2)^{\frac{n}{2}}}$
69. $\int \frac{u^m du}{(a^2 - u^2)^{\frac{n}{2}}} = \frac{u^{m+1}}{a^2(n-2)(a^2 - u^2)^{\frac{n}{2}-1}} - \frac{m-n+3}{a^2(n-2)} \int \frac{u^m du}{(a^2 - u^2)^{\frac{n}{2}-1}}$
70. $\int \frac{du}{u(a^2 - u^2)^{\frac{1}{2}}} = -\frac{1}{a} \ln \left(\frac{a + \sqrt{a^2 - u^2}}{u} \right) + C = -\frac{1}{a} \operatorname{arg} \coth \frac{a}{u} + C$
71. $\int \frac{du}{u^2(a^2 - u^2)^{\frac{1}{2}}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$
72. $\int \frac{du}{u^3(a^2 - u^2)^{\frac{1}{2}}} = -\frac{\sqrt{a^2 - u^2}}{2a^2 u^2} - \frac{1}{2a^3} \ln \left(\frac{a + \sqrt{a^2 - u^2}}{u} \right) + C = -\frac{\sqrt{a^2 - u^2}}{2a^2 u^2} - \frac{1}{2a^3} \operatorname{arg} \coth \frac{a}{u} + C$
73. $\int \frac{du}{u^m(a^2 - u^2)^{\frac{n}{2}}} = -\frac{1}{a^2(m-1)u^{m-1}(a^2 - u^2)^{\frac{n}{2}-1}} + \frac{m+n-3}{a^2(m-1)} \int \frac{du}{u^{m-2}(a^2 - u^2)^{\frac{n}{2}}}$

$$74. \int \frac{du}{u^m (a^2 - u^2)^{\frac{n}{2}}} = \frac{1}{a^2(n-2)u^{m-1}(a^2 - u^2)^{\frac{n}{2}-1}} + \frac{m+n-3}{a^2(n-2)} \int \frac{du}{u^m (a^2 - u^2)^{\frac{n}{2}-1}}$$

$$75. \int \frac{(a^2 - u^2)^{\frac{1}{2}} du}{u} = \sqrt{a^2 - u^2} - a \ln \left(\frac{a + \sqrt{a^2 - u^2}}{u} \right) + C = \sqrt{a^2 - u^2} - a \operatorname{arg} \coth \frac{a}{u} + C$$

$$76. \int \frac{(a^2 - u^2)^{\frac{1}{2}} du}{u^2} = -\frac{\sqrt{a^2 - u^2}}{u} - \arcsin \frac{u}{a} + C$$

$$77. \int \frac{(a^2 - u^2)^{\frac{n}{2}} du}{u^m} = -\frac{(a^2 - u^2)^{\frac{n}{2}+1}}{a^2(m-1)u^{m-1}} + \frac{m-n-3}{a^2(m-1)} \int \frac{(a^2 - u^2)^{\frac{n}{2}} du}{u^{m-2}}$$

$$78. \int \frac{(a^2 - u^2)^{\frac{n}{2}} du}{u^m} = \frac{(a^2 - u^2)^{\frac{n}{2}}}{(n-m+1)u^{m-1}} + \frac{a^2 n}{n-m+1} \int \frac{(a^2 - u^2)^{\frac{n}{2}-1} du}{u^m}$$

$\sqrt{2au \pm u^2}$ içeren ifadeler

$\sqrt{2au \pm u^2} = u^{1/2}(2a \pm u)^{1/2}$ eşitliği sayesinde 96-104 numaralı formüller de kullanılabilir.

$$79. \int \sqrt{2au - u^2} du = \frac{u-a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \arccos \left(1 - \frac{u}{a} \right) + C$$

$$80. \int u \sqrt{2au - u^2} du = -\frac{3a^2 + au - 2u^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \arccos \left(1 - \frac{u}{a} \right) + C$$

$$81. \int u^m \sqrt{2au - u^2} du = -\frac{u^{m-1}(2au - u^2)^{\frac{3}{2}}}{m+2} + \frac{a(2m+1)}{m+2} \int u^{m-1} \sqrt{2au - u^2} du$$

$$82. \int \frac{\sqrt{2au - u^2} du}{u} = \sqrt{2au - u^2} + a \arccos \left(1 - \frac{u}{a} \right) + C$$

$$83. \int \frac{\sqrt{2au - u^2} du}{u^2} = -\frac{2\sqrt{2au - u^2}}{u} - \arccos \left(1 - \frac{u}{a} \right) + C$$

$$84. \int \frac{\sqrt{2au - u^2} du}{u^3} = -\frac{(2au - u^2)^{\frac{3}{2}}}{3au^3} + C$$

$$85. \int \frac{\sqrt{2au - u^2} du}{u^m} = -\frac{(2au - u^2)^{\frac{3}{2}}}{a(2m-3)u^m} + \frac{m-3}{a(2m-3)} \int \frac{\sqrt{2au - u^2} du}{u^{m-1}}$$

$$86. \int \frac{du}{\sqrt{2au - u^2}} = \arccos \left(1 - \frac{u}{a} \right) + C$$

$$87. \int \frac{du}{\sqrt{2au + u^2}} = \ln \left(u + a + \sqrt{2au + u^2} \right) + C$$

$$88. \int F \left(u, \sqrt{2au + u^2} \right) du = \int F \left(z - a, \sqrt{z^2 - a^2} \right) dz \quad (\text{burada } z = u + a)$$

$$89. \int \frac{u du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \arccos \left(1 - \frac{u}{a} \right) + C$$

$$90. \int \frac{u^2 du}{\sqrt{2au-u^2}} = -\frac{(u+3a)\sqrt{2au-u^2}}{2} + \frac{3a^2}{2} \arccos\left(1-\frac{u}{a}\right) + C$$

$$91. \int \frac{u^m du}{\sqrt{2au-u^2}} = -\frac{u^{m-1}\sqrt{2au-u^2}}{m} + \frac{a(2m-1)}{m} \int \frac{u^{m-1} du}{\sqrt{2au-u^2}}$$

$$92. \int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$$

$$93. \int \frac{du}{u^m\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{a(2m-1)u^m} + \frac{m-1}{a(2m-1)} \int -\frac{du}{u^{m-1}\sqrt{2au-u^2}}$$

$$94. \int \frac{du}{(2au-u^2)^{\frac{3}{2}}} = \frac{u-a}{a^2\sqrt{2au-u^2}} + C$$

$$95. \int \frac{u du}{(2au-u^2)^{\frac{3}{2}}} = \frac{u}{a\sqrt{2au-u^2}} + C$$

Binomlar

$$96. \int u^m(a+bu^q)^p du = \frac{u^{m-q+1}(a+bu^q)^{p+1}}{b(pq+m+1)} - \frac{a(m-q+1)}{b(pq+m+1)} \int u^{m-q}(a+bu^q)^p du$$

$$97. \int u^m(a+bu^q)^p du = \frac{u^{m+1}(a+bu^q)^p}{pq+m+1} + \frac{apq}{pq+m+1} \int u^m(a+bu^q)^{p-1} du$$

$$98. \int \frac{du}{u^m(a+bu^q)^p} = -\frac{1}{a(m-1)u^{m-1}(a+bu^q)^{p-1}} - \frac{b(m-q+pq-1)}{a(m-1)} \int \frac{du}{u^{m-q}(a+bu^q)^p}$$

$$99. \int \frac{du}{u^m(a+bu^q)^p} = \frac{1}{aq(p-1)u^{m-1}(a+bu^q)^{p-1}} + \frac{m-q+pq-1}{aq(p-1)} \int \frac{du}{u^m(a+bu^q)^{p-1}}$$

$$100. \int \frac{du}{u(a+bu^q)} = \frac{1}{aq} \ln\left(\frac{u^q}{a+bu^q}\right) + C$$

$$101. \int \frac{(a+bu^q)^p du}{u^m} = -\frac{(a+bu^q)^{p+1}}{a(m-1)u^{m-1}} - \frac{b(m-q-pq-1)}{a(m-1)} \int \frac{(a+bu^q)^p du}{u^{m-q}}$$

$$102. \int \frac{(a+bu^q)^p du}{u^m} = \frac{(a+bu^q)^p}{(pq-m+1)u^{m-1}} + \frac{apq}{pq-m+1} \int \frac{(a+bu^q)^{p-1} du}{u^m}$$

$$103. \int \frac{u^m du}{(a+bu^q)^p} = \frac{u^{m-q+1}}{b(m-pq+1)(a+bu^q)^{p-1}} - \frac{a(m-q+1)}{b(m-pq+1)} \int \frac{u^{m-q} du}{(a+bu^q)^p}$$

$$104. \int \frac{u^m du}{(a+bu^q)^p} = \frac{u^{m+1}}{aq(p-1)(a+bu^q)^{p-1}} - \frac{m+q-pq+1}{aq(p-1)} \int \frac{u^m du}{(a+bu^q)^{p-1}}$$

$$90. \int \frac{u^2 du}{\sqrt{2au-u^2}} = -\frac{(u+3a)\sqrt{2au-u^2}}{2} + \frac{3a^2}{2} \arccos\left(1-\frac{u}{a}\right) + C$$

$$91. \int \frac{u^m du}{\sqrt{2au-u^2}} = -\frac{u^{m-1}\sqrt{2au-u^2}}{m} + \frac{a(2m-1)}{m} \int \frac{u^{m-1} du}{\sqrt{2au-u^2}}$$

$$92. \int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$$

$$93. \int \frac{du}{u^m \sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{a(2m-1)u^m} + \frac{m-1}{a(2m-1)} \int \frac{du}{u^{m-1} \sqrt{2au-u^2}}$$

$$94. \int \frac{du}{(2au-u^2)^{\frac{3}{2}}} = \frac{u-a}{a^2 \sqrt{2au-u^2}} + C$$

$$95. \int \frac{u du}{(2au-u^2)^{\frac{3}{2}}} = \frac{u}{a\sqrt{2au-u^2}} + C$$

Binomlar

$$96. \int u^m (a+bu^q)^p du = \frac{u^{m-q+1}(a+bu^q)^{p+1}}{b(pq+m+1)} - \frac{a(m-q+1)}{b(pq+m+1)} \int u^{m-q}(a+bu^q)^p du$$

$$97. \int u^m (a+bu^q)^p du = \frac{u^{m+1}(a+bu^q)^p}{pq+m+1} + \frac{apq}{pq+m+1} \int u^m (a+bu^q)^{p-1} du$$

$$98. \int \frac{du}{u^m (a+bu^q)^p} = -\frac{1}{a(m-1)u^{m-1}(a+bu^q)^{p-1}} - \frac{b(m-q+pq-1)}{a(m-1)} \int \frac{du}{u^{m-q}(a+bu^q)^p}$$

$$99. \int \frac{du}{u^m (a+bu^q)^p} = \frac{1}{aq(p-1)u^{m-1}(a+bu^q)^{p-1}} + \frac{m-q+pq-1}{aq(p-1)} \int \frac{du}{u^m (a+bu^q)^{p-1}}$$

$$100. \int \frac{du}{u(a+bu^q)} = \frac{1}{aq} \ln\left(\frac{u^q}{a+bu^q}\right) + C$$

$$101. \int \frac{(a+bu^q)^p du}{u^m} = -\frac{(a+bu^q)^{p+1}}{a(m-1)u^{m-1}} - \frac{b(m-q-pq-1)}{a(m-1)} \int \frac{(a+bu^q)^p du}{u^{m-q}}$$

$$102. \int \frac{(a+bu^q)^p du}{u^m} = \frac{(a+bu^q)^p}{(pq-m+1)u^{m-1}} + \frac{apq}{pq-m+1} \int \frac{(a+bu^q)^{p-1} du}{u^m}$$

$$103. \int \frac{u^m du}{(a+bu^q)^p} = \frac{u^{m-q+1}}{b(m-pq+1)(a+bu^q)^{p-1}} - \frac{a(m-q+1)}{b(m-pq+1)} \int \frac{u^{m-q} du}{(a+bu^q)^p}$$

$$104. \int \frac{u^m du}{(a+bu^q)^p} = \frac{u^{m+1}}{aq(p-1)(a+bu^q)^{p-1}} - \frac{m+q-pq+1}{aq(p-1)} \int \frac{u^m du}{(a+bu^q)^{p-1}}$$

$$119. \int \frac{du}{\sqrt{(u-a)(b-u)}} = 2 \arcsin \sqrt{\frac{u-a}{b-a}} + C$$

Eksp üstel ve logaritmik ifadeler

$$120. \int e^{au} du = \frac{e^{au}}{a} + C$$

$$121. \int b^{au} du = \frac{b^{au}}{a \ln(b)} + C$$

$$122. \int u e^{au} du = \frac{e^{au}}{a^2} (au - 1) + C$$

$$123. \int u^n e^{au} du = \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$124. \int u^n b^{au} du = \frac{u^n b^{au}}{a \ln(b)} - \frac{n}{a \ln(b)} \int u^{n-1} b^{au} du$$

$$125. \int \frac{b^{au}}{u^n} = -\frac{b^{au}}{(n-1)u^{n-1}} + \frac{a \ln(b)}{n-1} \int \frac{b^{au}}{u^{n-1}} du$$

$$126. \int \ln(u) du = u \ln(u) - u + C$$

$$127. \int u^n \ln(u) du = u^{n+1} \left(\frac{\ln(u)}{n+1} - \frac{1}{(n+1)^2} \right) + C$$

$$128. \int u^m \ln^n(u) du = \frac{u^{m+1}}{m+1} \ln^n(u) - \frac{n}{m+1} \int u^m \ln^{n-1}(u) du$$

$$129. \int e^{au} \ln(u) du = \frac{e^{au} \ln(u)}{a} - \frac{1}{a} \int \frac{e^{au}}{u} du$$

$$130. \int \frac{du}{u \ln(u)} = \ln(\ln(u)) + C$$

Trigonometrik İfadeler

$$131. \int \sin u du = -\cos u + C$$

$$132. \int \cos u du = \sin u + C$$

$$133. \int \tan u du = -\ln(\cos u) + C = \ln(\sec u) + C$$

$$134. \int \cot u du = \ln(\sin u) + C$$

$$135. \int \sec u du = \int \frac{du}{\cos u} = \ln(\sec u + \tan u) + C = \ln \left(\tan \left(\frac{u}{2} + \frac{\pi}{4} \right) \right) + C$$

$$136. \int \operatorname{cosec} u du = \int \frac{du}{\sin u} = \ln(\operatorname{cosec} u - \cot u) + C = \ln \left(\tan \frac{u}{2} \right) + C$$

$$137. \int \sec^2 u du = \tan u + C$$

$$138. \int \operatorname{cosec}^2 u du = -\cot u + C$$

$$139. \int \sec u \tan u du = \sec u + C$$

$$140. \int \operatorname{cosec} u \cot u \, du = -\operatorname{cosec} u + C$$

$$141. \int \sin^2 u \, du = \frac{u}{2} - \frac{1}{4} \sin 2u + C$$

$$142. \int \cos^2 u \, du = \frac{u}{2} + \frac{1}{4} \sin 2u + C$$

$$143. \int \cos^n u \sin u \, du = -\frac{\sin^{n+1} u}{n+1} + C$$

$$144. \int \sin^n u \cos u \, du = \frac{\sin^{n+1} u}{n+1} + C$$

$$145. \int \sin mu \sin nu \, du = -\frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$$

$$146. \int \cos mu \cos nu \, du = \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$$

$$147. \int \sin mu + \cos nu \, du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$$

$$148. \int \frac{du}{1 + \cos a \cos u} = 2 \operatorname{cosec} a \arctan \left(\tan \frac{a}{2} \tan \frac{u}{2} \right) + C$$

$$149. \int \frac{du}{\cos a + \cos u} = \operatorname{cosec} a \ln \left(\frac{1 + \tan \frac{a}{2} \tan \frac{u}{2}}{1 - \tan \frac{a}{2} \tan \frac{u}{2}} \right) + C = 2 \operatorname{cosec} a \operatorname{arctanh} \left(\tan \frac{a}{2} \tan \frac{u}{2} \right) + C$$

öyle ki $\tan^2 \frac{u}{2} < \cot^2 \frac{a}{2}$

$$150. \int \frac{du}{1 + \cos a \sin u} = 2 \operatorname{cosec} a \arctan \left(\operatorname{cosec} a \tan \frac{u}{2} + \cot a \right) + C$$

$$151. \int \frac{du}{\cos a + \sin u} = \operatorname{cosec} a \ln \left(\frac{\tan a - \tan \frac{u}{2} - \sec a}{\tan a + \tan \frac{u}{2} + \sec a} \right) + C = -2 \operatorname{cosec} a \operatorname{arctanh} \left(\cot a \tan \frac{u}{2} + \operatorname{cosec} a \right) + C$$

öyle ki $\left(\cot a \tan \frac{u}{2} + \operatorname{cosec} a \right)^2 < 1$

$$152. \int \frac{du}{a^2 \cos^2 u + b^2 \sin^2 u} = \frac{1}{ab} \arctan \left(\frac{b \tan u}{a} \right) + C$$

$$153. \int e^{au} \sin nu \, du = \frac{e^{au} (a \sin nu - n \cos nu)}{a^2 + n^2} + C$$

$$154. \int e^{au} \cos nu \, du = \frac{e^{au} (n \sin nu + a \cos nu)}{a^2 + n^2} + C$$

$$155. \int u \sin u \, du = \sin u - u \cos u + C$$

$$156. \int u \cos u \, du = \cos u + u \sin u + C$$

Trigonometrik ifadelerin tümevarımla integrali

$$157. \int \sin^n u \, du = -\frac{\sin^{n-1} u \cos u}{n} + \frac{n-1}{n} \int \sin^{n-2} u \, du$$

158. $\int \cos^n u \, du = \frac{\cos^{n-1} u \sin u}{n} + \frac{n-1}{n} \int \cos^{n-2} u \, du$
159. $\int \frac{du}{\sin^n u} = -\frac{\cos u}{(n-1)\sin^{n-1} u} + \frac{n-2}{n-1} \int \frac{du}{\sin^{n-2} u}$
160. $\int \frac{du}{\cos^n u} = \frac{\sin u}{(n-1)\cos^{n-1} u} + \frac{n-2}{n-1} \int \frac{du}{\cos^{n-2} u}$
161. $\int \cos^m u \sin^n u \, du = \frac{\cos^{m-1} u \sin^{n+1} u}{m+n} + \frac{m-1}{m+n} \int \cos^{m-2} u \sin^n u \, du$
162. $\int \cos^m u \sin^n u \, du = -\frac{\sin^{n-1} u \cos^{m+1} u}{m+n} + \frac{n-1}{m+n} \int \cos^m u \sin^{n-2} u \, du$
163. $\int \frac{du}{\cos^m u \sin^n u} = \frac{1}{(m-1)\sin^{n-1} u \cos^{m-1} u} + \frac{m+n-2}{m-1} \int \frac{du}{\cos^{m-2} u \sin^n u}$
164. $\int \frac{du}{\cos^m u \sin^n u} = -\frac{1}{(n-1)\sin^{n-1} u \cos^{m-1} u} + \frac{m+n-2}{n-1} \int \frac{du}{\cos^m u \sin^{n-2} u}$
165. $\int \frac{\cos^m u \, du}{\sin^n u} = -\frac{\cos^{m+1} u}{(n-1)\sin^{n-1} u} - \frac{m-n+2}{n-1} \int \frac{\cos^m u \, du}{\sin^{n-2} u}$
166. $\int \frac{\cos^m u \, du}{\sin^n u} = \frac{\cos^{m-1} u}{(m-n)\sin^{n-1} u} + \frac{m-1}{m-n} \int \frac{\cos^{m-2} u \, du}{\sin^n u}$
167. $\int \frac{\sin^n u \, du}{\cos^m u} = \frac{\sin^{n+1} u}{(m-1)\cos^{m-1} u} - \frac{n-m+2}{m-1} \int \frac{\sin^n u \, du}{\cos^{m-2} u}$
168. $\int \frac{\sin^n u \, du}{\cos^m u} = -\frac{\sin^{n-1} u}{(n-m)\cos^{m-1} u} + \frac{n-1}{n-m} \int \frac{\sin^{n-2} u \, du}{\cos^m u}$
169. $\int \tan^n u \, du = \frac{\tan^{n-1} u}{n-1} - \int \tan^{n-2} u \, du$
170. $\int \cot^n u \, du = -\frac{\cot^{n-1} u}{n-1} - \int \cot^{n-2} u \, du$
171. $\int e^{au} \cos^n u \, du = \frac{e^{au} \cos^{n-1} u (a \cos u + n \sin u)}{a^2 + n^2} + \frac{n(n-1)}{a^2 + n^2} \int e^{au} \cos^{n-2} u \, du$
172. $\int e^{au} \sin^n u \, du = \frac{e^{au} \sin^{n-1} u (a \sin u - n \cos u)}{a^2 + n^2} + \frac{n(n-1)}{a^2 + n^2} \int e^{au} \sin^{n-2} u \, du$
173. $\int u^m \cos au \, du = \frac{u^{m-1}}{a^2} (au \sin au + m \cos au) - \frac{m(m-1)}{a^2} \int u^{m-2} \cos au \, du$
174. $\int u^m \sin au \, du = \frac{u^{m-1}}{a^2} (m \sin au - au \cos au) - \frac{m(m-1)}{a^2} \int u^{m-2} \sin au \, du$

Ters trigonometrik ifadeler

175. $\int \arcsin u \, du = u \arcsin u + \sqrt{1-u^2} + C$
176. $\int \arccos u \, du = u \arccos u - \sqrt{1-u^2} + C$
177. $\int \arctan u \, du = u \arctan u - \ln(\sqrt{1+u^2}) + C$
178. $\int \operatorname{arccot} u \, du = u \operatorname{arccot} u + \ln(\sqrt{1+u^2}) + C$

$$179. \int \operatorname{arcsec} u \, du = u \operatorname{arcsec} u - \ln\left(u + \sqrt{u^2 - 1}\right) + C = u \operatorname{arcsec} u - \arg \cosh u + C$$

$$180. \int \operatorname{arccosec} u \, du = u \operatorname{arccosec} u + \ln\left(u + \sqrt{u^2 - 1}\right) + C = u \operatorname{arccosec} u + \arg \cosh u + C$$

Hiperbolik fonksiyonlar

$$181. \int \sinh u \, du = \cosh u + C$$

$$182. \int \cosh u \, du = \sinh u + C$$

$$183. \int \tanh u \, du = \ln(\cosh u) + C$$

$$184. \int \coth u \, du = \ln(\sinh u) + C$$

$$185. \int \operatorname{sech} u \, du = \arctan(\sinh u) + C = \operatorname{gd} u + C$$

$$186. \int \operatorname{cosec} u \, du = \ln\left(\tanh \frac{u}{2}\right) + C$$

$$187. \int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$188. \int \operatorname{cosech}^2 u \, du = -\coth u + C$$

$$197. \int \arg \sinh u \, du = u \arg \sinh u - \sqrt{1 + u^2} + C$$

$$198. \int \arg \cosh u \, du = u \arg \cosh u - \sqrt{u^2 - 1} + C$$

$$199. \int \arg \tanh u \, du = u \arg \tanh u + \frac{1}{2} \ln(1 - u^2) + C$$

$$200. \int \arg \coth u \, du = u \arg \coth u + \frac{1}{2} \ln(1 - u^2) + C$$

$$201. \int \arg \operatorname{sech} u \, du = u \arg \operatorname{sech} u + \operatorname{gd}(\arg \tanh u) + C = u \arg \operatorname{sech} u + \arcsin u + C$$

$$202. \int \arg \operatorname{cosech} u \, du = u \arg \operatorname{cosech} u + \arg \sinh u + C$$

$$203. \int \sinh mu \sinh nu \, du = \frac{\sinh(m+n)u}{2(m+n)} - \frac{\sinh(m-n)u}{2(m-n)} + C$$

$$204. \int \cosh mu \cosh nu \, du = \frac{\sinh(m+n)u}{2(m+n)} + \frac{\sinh(m-n)u}{2(m-n)} + C$$

$$205. \int \sinh mu \cosh nu \, du = \frac{\cosh(m+n)u}{2(m+n)} + \frac{\cosh(m-n)u}{2(m-n)} + C$$

$$206. \int \frac{du}{\cosh a + \cosh u} = 2 \operatorname{cosech} a \cdot \arg \tanh\left(\tanh \frac{u}{2} \tanh \frac{a}{2}\right) + C$$

$$207. \int \frac{du}{\cos a + \cosh u} = 2 \operatorname{cosec} a \cdot \arg \tan\left(\tanh \frac{u}{2} \tan \frac{a}{2}\right) + C$$

$$208. \int \frac{du}{1 + \cos a \cosh u} = 2 \operatorname{cosec} a \cdot \arg \tanh\left(\tanh \frac{u}{2} \tan \frac{a}{2}\right) + C \quad \text{eğer } \tan^2 \frac{u}{2} < \cot^2 \frac{a}{2} \text{ ise}$$

$$209. \int e^{au} \sinh nu \, du = \frac{e^{au}(a \sinh nu - n \cosh nu)}{a^2 - n^2} + C$$

$$210. \int e^{au} \cosh nu \, du = \frac{e^{au}(a \cosh nu - n \sinh nu)}{a^2 - n^2} + C$$

$$189. \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$190. \int \operatorname{cosech} u \coth u \, du = -\operatorname{cosech} u + C$$

$$191. \int \sinh^2 u \, du = \frac{1}{4} \sinh 2u - \frac{u}{2} + C$$

$$192. \int \cosh^2 u \, du = \frac{1}{4} \sinh 2u + \frac{u}{2} + C$$

$$193. \int \tanh^2 u \, du = u - \tanh u + C$$

$$194. \int \coth^2 u \, du = u - \coth u + C$$

$$195. \int u \sinh u \, du = u \cosh u - \sinh u + C$$

$$196. \int u \cosh u \, du = u \sinh u - \cosh u + C$$

Kaynakça

Granville, Smith ve Longley, *Éléments de Calcul Différentiel et Intégral*, Vubert 1970.