

Math 151
Midterm
November 2004
Ali Nesin

Do not use symbols like \exists , \forall , \Rightarrow . Make full and precise sentences.

1. Find $\lim_{n \rightarrow \infty} \frac{2n-5}{3n+4}$ and prove your result by using the definition of convergence.
2. Show that if $a_n \leq a_{n+1} \leq b_{n+1} \leq b_n$ for all n , then $\bigcap_{n \in \mathbb{N}} [a_n, b_n]$ is a nonempty interval.
3. Let $x_1 = 1$, $x_2 = 2$ and $x_n = (x_{n-1} + x_{n-2})/2$ for $n > 2$.
 - 3a. Show that $1 \leq x_n \leq 2$ for all n .
 - 3b. Show that $|x_n - x_{n+1}| = 1/2^{n-1}$ for all n .
 - 3c. Show that if $m > n$ then $|x_n - x_m| < 1/2^{n-2}$ for all n .
 - 3d. Show that $(x_n)_n$ is a Cauchy sequence.
 - 3e. Find its limit.
4. We say that a sequence $(x_n)_n$ is **contractive** if there is a constant c , $0 < c < 1$, such that $|x_{n+2} - x_{n+1}| \leq c |x_{n+1} - x_n|$ for all n . Show that every contractive sequence is convergent.