

Math 151

Exercises (quiz)

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Justify all your answers. A nonjustified answer will not receive any grade whatsoever, even if the answer is correct. DO NOT use symbols such as \forall , \exists , \Rightarrow . Make full sentences with correct punctuation.

- Let $(a_n)_n$ be a convergent sequence of real numbers.
 - Does the sequence $(a_{2n})_n$ converge necessarily?
 - Assume $a_n \neq 0$ for all n . Does the sequence $(a_n/a_{n+1})_n$ converge necessarily?
- Let $(a_n)_n$ be a convergent sequence of real numbers. Suppose that $a_n \in \mathbb{Z}$ for all n . Prove or disprove: $\lim_{n \rightarrow \infty} a_n \in \mathbb{Z}$.
- Let $(a_n)_n$ be a convergent sequence of real numbers. Suppose that $5a_n/2 \in \mathbb{N}$ for all n . What can you say about $\lim_{n \rightarrow \infty} a_n$?
- Let $(a_n)_n$ be a sequence of real numbers such that the sequence $(a_n^2)_n$ converges to 0. Does the sequence $(a_n)_n$ converge necessarily?
- Find the following limits and prove your result using only the definition.
 - $\lim_{n \rightarrow \infty} \frac{2n-5}{5n+2}$
 - $\lim_{n \rightarrow \infty} \frac{2n^2-5}{-5n+2}$
 - $\lim_{n \rightarrow \infty} \frac{2n^2-5}{n^3+2}$

Note: A sequence $(a_n)_n$ of real numbers is said to *converge to infinity* if for all A there is an N such that if $n > N$ then $a_n > A$.
- Let $(a_n)_n$ be a sequence of real numbers such that $\lim_{n \rightarrow \infty} a_n = \infty$. Show that $\lim_{n \rightarrow \infty} a_{2n} = \infty$?
- Let $(a_n)_n$ be a sequence of nonnegative real numbers. Suppose that the sequence $(a_n^2)_n$ converges to a . Show that the sequence $(a_n)_n$ converges to \sqrt{a} .