MATH 211 Basic Algebra

Problem Set 5

1. Let the orbits of $\sigma \in S_n$ have sizes $n_1, \ldots, n_k$. Find the order of σ.
2. What is the order of the permutation $i \mapsto i+1$ in in $\mathrm{Sym}(\mathbb{Z})$?
3. Prove that in S_n the order of any odd permutation is an even number.
4. What is the order of the elements $(-\sqrt{3}+i)/2$ and $(3+4i)/5$ in $\mathbb{C}^*$?
5. What is the order in the group $T_2(\mathbb{C})$ of the elements
$$\begin{pmatrix} -1 & w \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} z & 0 \\ 0 & u \end{pmatrix},$$
where $z, u \in \mathbb{C}^*$, $w \in \mathbb{C}$.
6. How many elements of order 6 are there in $\mathbb{C}^*$, $D_2(\mathbb{C})$, S_5, A_5?
7. Show that for any $n, k, m \in \{0, 1, \ldots, \infty\}$ in $\mathrm{Sym}(\mathbb{N})$ there are elements σ, π such that $\mathrm{order}(\sigma) = n$, $\mathrm{order}(\pi) = k$, and $\mathrm{order}(\sigma\pi) = m$.
8. Prove that in any group elements x and $y^{-1}xy$ have the same order.
9. Prove that in any group elements ab and ba have the same order.
10. Suppose x, y have finite orders n, m in a group G, and $xy = yx$. Show that xy has finite order. Prove that if m and n are coprime then the order of xy is nm. Show that the condition "m and n are coprime" is essential.
11. What is the order of a^k if the order of a is n?
12. How many subgroups are there in the cyclic group of order 100? 81? p^n, where p is prime?
13. Prove that the group $\mathbb{Q}$ is not cyclic.
14. Find all subgroups of $\mathbb{C}(p^\infty)$.
15. Prove that any finite subgroup of $\mathbb{C}^*$ is cyclic.