

**MATH 312**  
**RING THEORY**  
**Final exam**  
**Prof. Oleg Belegradek**

1. Prove that the sum of two nilpotent right ideals is a nilpotent right ideal.
2. (a) Prove that in any ring every strictly nilpotent element is nilpotent.  
(b) Give an example of a ring in which there is a nilpotent element which is not strictly nilpotent.  
(c) Prove that in a commutative ring any nilpotent element is strictly nilpotent.
3. Prove that if  $M$  is an irreducible  $R$ -module then the ring  $R/\text{Ann}(M)$  is primitive.
4. Prove that an artinian ring without zero divisors is a division ring.
5. Prove that a ring  $R$  is local iff there is an ideal  $I$  of  $R$  such that  $R/I$  is a division ring.
6. Prove that any noetherian nil ring is nilpotent.