

Algebra (Math 211) Final

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- 1a. How many conjugacy classes are there in $\text{Sym}(5)$?
- 1b. Find the sizes of the centralizers of the elements of $\text{Sym}(5)$.

- 2a. Find all automorphisms of the additive group of \mathbf{Z} .
- 2b. Find all automorphisms of the ring \mathbf{Q} .

- 3a. Find a generator of the subgroup of \mathbf{Q}^* generated by $3/5$ and $5/7$.
- 3b. Is the subgroup of \mathbf{Q}^+ generated by $\{3^n : n \in \mathbf{Z}\}$ cyclic?
- 3c. Is the subgroup of \mathbf{Q}^* generated by $\{3^n : n \in \mathbf{Z}\}$ cyclic?

- 4a. Find three elements of order 2 of $\mathbf{R}^*/\langle\sqrt{12}\rangle$.
- 4b. Is there an element of infinite order in \mathbf{Q}/\mathbf{Z} ? Justify your answer.
- 4c. Is there an element of finite order in \mathbf{R}/\mathbf{Q} ? Justify your answer.

5. Show that the set of elements σ of $\text{Sym}(\mathbf{Z})$ such that $\sigma(0) = 0$ is a subgroup isomorphic to $\text{Sym}(\mathbf{Z})$.

6. Find a nonabelian group of order 8.

7. Find the invertible elements of $\mathbf{R}[X, Y]/I$ where I is the principal ideal generated by $X^2 - Y^2$.

8. Let R be a ring (not necessarily commutative, does not necessarily have an identity) where $x^2 = x$ for all $x \in R$. Show that R is commutative and has characteristic 2.

9. Is the ideal generated by $X - Y$ a maximal ideal of $\mathbf{R}[X, Y]$? Justify your answer.

10. Find a maximal ideal of $\mathbf{Z}[X, Y]/I$ where I is the ideal generated by $X^3 - Y^2$.